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STEP MODEL OF EROSION OF ELECTRODES. I. APPLICATION TO ARC SPOTS ON THE CATHODES OF ELECTRIC-ARC HEATERS

A. Marotta,[a] **L. I. Sharakhovskii,**[b] **and A. M. Esipchuk**[b]

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A simple model of erosion of cold electrodes which considers the step motion of arc spots is proposed. It is assumed that the macroscopic destruction of an electrode begins with fusion of the electrode in the spot. The basic thermal characteristics of the arc spot, the thermophysical properties of the electrode material, and the regime of the electric arc are taken into account. The notion of the effective erosion enthalpy which summarizes all the elementary processes leading to the erosion of the electrode is used. The model has successfully been applied to arc spots on the cathodes of electric-arc heaters. A comparison with the earlier model of continuous motion of the spot has been made.

Introduction. The problem of erosion of electrodes is pressing for a number of applications of an electric arc because of the limited service life of plants, in particular, electric-arc heaters and electric switches. In electric-arc heaters, the process of transfer of a current between the arc and the cold electrodes manufactured from materials with a low fusion temperature, for example, from copper, is carried out in the contracted regime. Virtually the entire current traverses a bounded region on the electrode surface which is characterized by a very high density of the current and the heat flux and is known as the arc spot. To decrease erosion in electric-arc heaters with cold electrodes one uses nonstationary spots, causing them to move rapidly by means of a magnetic field or a vortex gas spot. The most important is investigation of the erosion of a cold cathode since it is higher than that in the anode.

Owing to its importance, the issue of the density of a current in a cathode spot remains the subject of continuous discussions. The data on the current density in cathode spots on copper electrodes are very uncertain and contradictory because of the large spread — from 10^7 to 10^{12} $A{\cdot}m^{-2}$ (see, for example, [1–3]). The use of equipment with a high temporal and spatial resolution [1–3] has made it possible to investigate the internal structure of a spot consisting of short-lived mobile coexisting arc fragments or microspots with a dimension of less that 1 μm. Moreover, with development of the technology of high-speed optical recording, one discloses increasingly smaller details of the complex hierarchic microstructure of a cathode spot. Some of the modern experimental data and theoretical results lead to current densities of 10^{12} to 10^{13} $A{\cdot}m^{-2}$ in cathode microspots, whereas the data time- and space-averaged using instrumental methods yield a value of about 10^9 $A{\cdot}m^{-2}$ (see, for example, [1]).

Despite the complex dynamic internal structure of a spot, one can apply to engineering calculations the replacement (used earlier (see [4, 5])) of the actual arc spot by a single surface heat source with a uniform distribution of the heat flux which is obtained by averaging the thermal fields of individual microspots over space and time: $q_0 = jU = 4Q_0/\pi d^2$. For this reason, here we call j and U the "effective" current density (instead of the real current density) and the volt-equivalent of the heat flux in the arc spot ($U = Q_0/I$) respectively.

The continuous movement of an arc spot considered in [4] can be ensured for each type of electrode material only by a specially selected chemical composition of a plasma-forming gas (see [6] and the references therein). In [6], it has been assumed that there exists special "surface resistance" which leads to a step-by-step (with stops) movement of an arc spot and as a consequence to an increase in the erosion due to the increased time of residence of the spot

[a]Gleb Wataghin Institute of Physics at Campinas State University, Campinas, Brazil (Instituto de Fisica "Gleb Wataghin," Universidade Estadual de Campinas, Campinas, Sã Paulo, Brasil); email: aruy@ifi.unicamp.br; [b]A. V. Luikov Heat and Mass Transfer Institute, National Academy of Sciences of Belarus, Minsk, Belarus; email: leonids@land.ru. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 76, No. 2, pp. 116–122, March–April, 2003. Original article submitted October 14, 2002

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at the point of stop. Thus, the continuous movement of an arc spot can be considered as one ideal limiting regime. The other limiting regime is the regime (subsequently referred to as a "step" regime) of motion of the spot [7, 8]. In actual practice, the motion of the arc spot in an electric-arc heater has a complex character, as a rule, and it is a combination of both regimes. In the present work, attention is focused on studying the ideal step movement of the arc spot and its influence on the erosion of the electrodes in an electric-arc heater. This model can be applied to both the cathode and the anode of the electric-arc heater but for the sake of brevity here we will consider its application just to the cathode.

In [4], we have proposed a single parameter (called the effective enthalpy of erosion h_{ef}) to allow for all the processes leading to erosion; erosion is considered as the process of thermal ablation of the electrode material under the action of intense heat fluxes. This approach is analogous to that used in the technique of testing of materials for protection of spacecraft at their entry into the atmosphere [9, 10], where the resistance of a material to thermal damage is characterized by the effective enthalpy of ablation determined according to the equation

$$h_{ef} = \frac{Q_{in} - Q_{out}}{G} . \tag{1}$$

For a cold copper cathode in the electric-arc heater we disregard the removal of heat by radiation and the Joule heating (see [11]); instead, we allow for the removal of heat from the arc spot to the body of the electrode by conduction, taking that $Q_0 - Q_r = Q_{er}$.

Theoretical Approach. Similarly to [4, 5], we consider the heating of the cold-cathode surface by an arc spot, taking a uniformly distributed heat flux of density q_0 which is prescribed within a circularly shaped arc spot of diameter d (boundary conditions of the second kind according to [12, 13]). We assume that, according to [11], the time of stay of the spot at a given point τ_s satisfies the condition with respect to the Fourier number Fo = $a\tau_s/d^2$ = 1. Satisfaction of this condition enables us to use a one-dimensional heat-conduction equation for description of the heating of the electrode in the spot. The electrode is considered as a semibounded body, since $d \ll b$. Part of the mathematical formalism is the same as has been presented in [4, 5] for the model of continuous motion of the spot. Therefore, we will not repeat it here and begin with the equation that yields the density of the heat flux q_r removed from the arc spot to the body of the electrode after the beginning of fusion $\tau \ge \tau_0$ (see [4, 5] for details):

$$q_r = \frac{2q_0}{\pi} \arctan \sqrt{\frac{\tau_0}{\tau - \tau_0}} , \tag{2}$$

where

$$\tau_0 = \frac{\pi}{4a} \left[\frac{(T_f - T_0)\, \lambda}{q_0} \right]^2 \tag{3}$$

has been obtained in [4, 5].

We assume that, to the instant of time τ_0, in accordance with the boundary conditions of the second kind, the heat removal to the body of the electrode by conduction is equal to the heat supply from the arc spot ($q_r = q_0$ for $\tau < \tau_0$). Beginning with $\tau = \tau_0$ we assume that a constant fusion temperature $T = T_f$ is maintained on the surface above the spot, which corresponds to boundary conditions of the first kind (see [12, 13]). Using Eq. (2) we calculate the integral heat removal to the body of the electrode for $\tau_0 < \tau < \tau_s$.

Let us consider an ideal step motion of the spot, assuming that it exists only at discrete points on the electrode surface at a constant distance (equal to the step length L) between them. We will assume that the time of movement of the spot from one point of stop to another is negligibly small as compared to the time of stay of the spot at a fixed point τ_s. Then τ_s can be represented as

$$\tau_s = L/v . \tag{4}$$

The average velocity of movement of the arc spot can be obtained, for example, by measuring the rotational speed of the arc in the interelectrode gap.

To calculate the integral heat removal per unit area of the arc spot (in J·m^{-2}) from the instant of the beginning of fusion to an arbitrary instant of time τ_s we must integrate relation (2) with respect to time going from τ_0 to τ_s. For it we introduce the following notation:

$$q_r \Big|_{\tau_0}^{\tau_s} \equiv \frac{2q_0}{\pi} \int_{\tau_0}^{\tau_s} \arctan \sqrt{\frac{\tau_0}{\tau - \tau_0}}\, d\tau \,. \quad (5)$$

Integration of (5) leads to

$$q_r \Big|_{\tau_0}^{\tau_s} = \frac{2q_0}{\pi} \left[\frac{\pi}{2} (\tau_s - \tau_0) + \sqrt{\tau_0 (\tau_s - \tau_0)} - \tau_s \arctan \sqrt{\frac{\tau_s - \tau_0}{\tau_0}} \right]. \quad (6)$$

The total specific heat removal (in J·m^{-2}) per unit area of the arc spot from the beginning of heating to the instant of time τ_s, including the initial period $\tau < \tau_0$, is given by the relation

$$q_r \Big|_{\tau=0}^{\tau_s} = q_0 \tau_0 + q_r \Big|_{\tau_0}^{\tau_s} . \quad (7)$$

Substituting (6) into (7) and dividing the result by τ_s, we obtain the heat removal averaged over the time τ_s per unit area of the arc spot (in W·m^{-2}), which is equal to

$$\overline{q}_r = q_0 \left[1 + \frac{2}{\pi} \left(\frac{\sqrt{\tau_0 (\tau_s - \tau_0)}}{\tau_s} - \arctan \sqrt{\frac{\tau_s - \tau_0}{\tau_0}} \right) \right]. \quad (8)$$

In accordance with the determination of h_{ef}, in our case relation (1) can be written in the form

$$h_{ef} = \frac{Q_0 - \overline{Q}_r}{G} , \quad (9)$$

where $\overline{Q}_r = \overline{q}_r F$ is the total removal to the body of the electrode by heat conduction calculated over the entire area of the spot F and averaged over the residence time τ_s. Then, using Eq. (8), for $Q_0 = q_0 F$ we obtain

$$\overline{Q}_r = Q_0 \left[1 + \frac{2}{\pi} \left(\sqrt{f_s (1 - f_s)} \right) - \tan^{-1} \sqrt{\frac{1 - f_s}{f_s}} \right]. \quad (10)$$

Here we introduce the dimensionless parameter of fusion

$$f_s = \frac{\tau_0}{\tau_s} . \quad (11)$$

We will assume the difference between the total heat supply to the arc spot Q_0 and the total heat removal $\overline{Q}_r$ yields the total expenditure of heat on erosion. This heat determining the erosion rate will subsequently be called the "erosion heat" $\overline{Q}_{er}$:

$$\overline{Q}_{er} = Q_0 - \overline{Q}_r . \quad (12)$$

Substituting (10) into (12), we obtain that the average erosion heat can be expressed as follows:

$$\overline{Q}_{er} = Q_0 W_s = IUW_s , \quad (13)$$

where

$$W_s = \frac{2}{\pi}\left(\arctan\sqrt{\frac{1-f_s}{f_s}} - \sqrt{f_s - (1-f_s)}\right). \tag{14}$$

Using the expression for the mass rate of erosion $G = gI$, written in terms of the specific mass erosion g, and Eqs. (9), (12), and (13), we obtain the simple expression for the specific mass erosion

$$g = g_0 + \frac{UW_s}{h_{ef}}. \tag{15}$$

Here we have introduced the additional term g_0 to allow for the fact that, even in the absence of macroerosion when $W_s = 0$, we have the nonvanishing minimum value of the erosion $g = g_0$ observed experimentally. This residual erosion is attributable to the nonvanishing erosion action of microspots. As we will see subsequently in investigating the dependence of the erosion on the current, microerosion is observed at low currents, giving an approximately constant value of g_0. We note that

$$UW_s = (g - g_0)\, h_{ef}, \tag{16}$$

i.e., the group UW_s is a linear function of g. Plotting UW_s calculated from experimental data as a function of g measured experimentally on the graph, we can obtain the erosion parameters g_0 and h_{ef} just in the same manner as in [4, 5].

Thus, the main parameters determining erosion in the step model of erosion are the arc current I, the electrode-surface temperature T_0, and the residence time of the arc spot τ_s.

Substituting the expression for τ_0 from (3) into (11), for $d = 2\sqrt{I/\pi j}$ and $q_0 = jU$ we obtain the following relation:

$$f_s = \frac{\pi}{4a\tau_s}\left[\frac{(T_f - T_0)\,\lambda}{jU}\right]^2. \tag{17}$$

Using τ_s according to (4) and introducing the dimensionless step length n

$$n = L/d, \tag{18}$$

we have

$$f_s = \frac{\pi^{1.5} v \lambda^2 (T_f - T_0)^2}{8 a j^{1.5} U^2 I^{0.5} n} \tag{19}$$

or

$$f_s = \frac{\pi^{1.5} v \lambda^2 (T_f - T_0)^2}{8 a j_s^{1.5} U^2 I^{0.5}}, \tag{20}$$

where j_s as a new, imaginary current density for step motion is determined from the relation

$$j_s = j n^{2/3}. \tag{21}$$

Expression (17) will be useful in calculating the parameter f_s for immobile (pulse) arc spots ($v = 0$), for example, in electroerosion treatment, whereas Eqs. (19) and (20) will be useful in calculating moving arc spots in electric-arc heaters. We note that j here denotes the effective current density, which does not necessarily coincide with the density determined from the current-conducting area of the spot. Both j and j_s extend the notion of the ordinary current density,

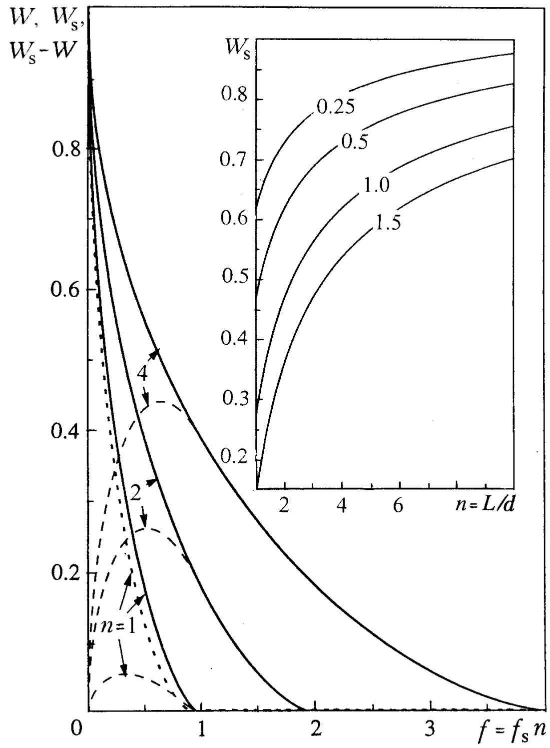

Fig. 1. Dimensionless energy of erosion W for the continuous (dotted line) and step motion W_s (solid lines) for different values of n (shown in the figure); dashed curves, $W_s - W$; inset, parameter W_s as a function of n; numbers of the curves, normalized velocity of the arc $s = vI^{-0.5}$.

taking into consideration primarily the thermal effect of the spot. Moreover, j_s also allows for the features of the step motion of the spot. Expressions (19) and (20), owing to the similarity of their form to that of the expression for f, are the most convenient for comparative analysis of the models of continuous and step spot motions.

The analysis of the function W_s shows that $1 \geq W_s \geq 0$ for $0 \leq f_s \leq 1$. We assume that $W_s \equiv 0$ for $f_s > 1$. Thus, we have two regions for f_s: the region of microerosion for $f_s > 1$ and the region of macroerosion for $0 \leq f_s \leq 1$. Recording simultaneously all the determining parameters involved in the expression for $f_s(I, T_0, v)$, for the regimes in which the condition $f_s = 1$ is fulfilled we can calculate the effective current density, using formula (17), i.e.,

$$j = \frac{\pi^{0.5} \lambda (T_f - T_0)}{2 a^{0.5} \tau_s^{0.5} U} \tag{22}$$

or

$$j = \frac{\pi}{4} \left[\frac{\lambda^4}{a^2} \frac{v^2 (T_f - T_0)^4}{U^4 I n^2} \right]^{1/3} . \tag{23}$$

Equation (22) will be useful in calculating the effective current density j in immobile spots ($v = 0$) in the processes of electroerosion treatment, whereas Eq. (23) will be useful in calculating moving spots in electric-arc heaters. From these equations we see that in step motion it is necessary to know τ_s or n for determination of the effective value of the current density j that is closer to the true value and not the "imaginary" value. In the case where n is unknown or one takes $n = 1$ in Eq. (23), we obtain only the imaginary current density related to the effective j by relation (21).

The dimensionless energy of erosion $W_s(f_s)$ plays the same role in the step model as the corresponding parameter $W(f)$ in the continuous-motion model. For further comparison of W_s and W we recall here the expression for W (see [4])

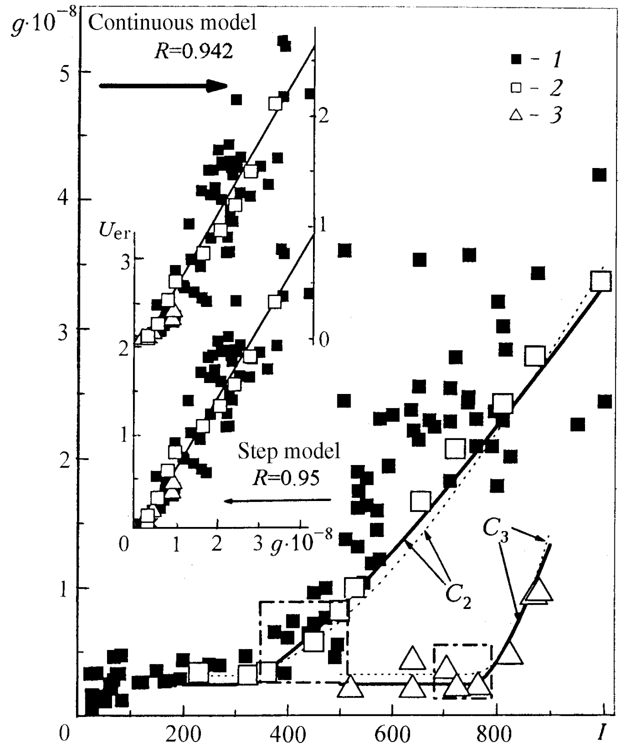

Fig. 2. Specific erosion rate g as a function of the arc current I: 1, 2) inside diameter of the cathode is 50 mm; 2, 3) "singular" points obtained for constant operating parameters [2) for a cathode diameter of 50 mm and B = 0.133 T, 3) 90 and 0.03]; C_2, calculated curve, according to Eqs. (14), (15), (19), and (24) for points 2; C_3, the same, for points 3; solid curves; step model, dotted curves, continuous-motion model according to [4]; inset, linear approximation of the parameter U_{er} vs. specific erosion g for the model of continuous and step motion.

$$W = \frac{2}{\pi}\left[\arcsin\beta + f\beta - \frac{4}{\pi}\beta\,(\sqrt{f}\,\omega_1 + \omega_2)\right], \tag{24}$$

where β, ω_1, and ω_2 are functions of the dimensionless parameter f which is coincident with f_s at $n = 1$ (see (19)).

Figure 1 shows W and W_s and the difference $W_s - W$ (dashed curves) as a function of f for different values of n. It is seen that erosion for the step motion of the spot is always higher than that for the continuous motion, except for the limiting points $f_s = 0$ or $f_s = 1$, where the difference is $W_s - W = 0$. Furthermore, this difference increases rapidly with n. The inset of Fig. 1 shows the dimensionless erosion energy W_s for a copper cathode at T_0 = 350 K as a function of the dimensionless step length for different values of the normalized velocity of the arc s (see [4, 5]) determined as $s = v/\sqrt{I}$. It is seen that the erosion grows with both increase in the dimensionless step length and decrease in the normalized velocity s. In the calculations, we used U = 6.78 V, $j = 1.35 \cdot 10^9$ $A \cdot m^{-2}$, λ = 377 $W \cdot m^{-1} \cdot K^{-1}$, $a = 1 \cdot 10^{-4}$ $m^2 \cdot sec^{-1}$, T_0 = 350 K, and T_f = 1356 K (see [4, 5]).

Comparison with Experiment for Electric-Arc Heaters. To compare the models of step and continuous motion of the spot we have used the experiments on investigating the erosion of a copper cathode [1]. Therefore, here we will describe in detail neither the experiments themselves nor the setup. We only note that this was a setup with a coaxial arrangement of the electrodes and magnetic movement of the arc without using a vortex gas flow.

Figure 2 gives results of experimental measurements of the specific mass erosion g as a function of the current I and (on the inset) the value of the volt-equivalent of the erosion energy $U_{er} = UW$ calculated for the same experiments for the models of step and continuous motion as a function of g (see Eq. (16)). The data on the current density are important for theoretical processing of these results. To obtain them we assumed that $f = 1$ for those experimental points in Fig. 2 (in the dash-dot frame) where the transition from the lower branch of low erosion to the upper branch of the intense form of erosion is observed. In this case the current density was calculated from formula (23), when $n = 1$, for the step motion. Since the formulas for j are the same for both models, when $n = 1$, in generalizing these experiments using the step model the only difference was in the expressions for the dimensionless energy of erosion W and W_s.

In processing these experiments (inset in Fig. 2), we used the same dependences of the current density and the volt-equivalent on the magnetic field as in [14, 15] in the model of continuous motion: $U = 6.25 + 4.28B$ and $j_s = (1.282 + 2.6B) \cdot 10^9$. It is obvious that the results of comparative processing of the same experiments are close for both models, but the correlation coefficient for the step model is slightly higher than that for the continuous model (0.95 instead of 0.94). From the linear approximations in the form $U_{er} = f(g)$ (see [4, 5] for the continuous model) which are shown in the inset, we obtained the values of q_0 and h_{ef}. They differ somewhat: $h_{ef} = 66$ MJ·kg^{-1} and $g_0 = 3.1 \cdot 10^{-9}$ kg·C^{-1} for the continuous model as compared to $h_{ef} = 81$ MJ·kg^{-1} and $g_0 = 2.44 \cdot 10^{-9}$ kg·C^{-1} for the step model. Using the quantities h_{ef} and g_0, we subsequently calculated the theoretical curves C_2 and C_3 shown in Fig. 2, where the solid line is the step model and the dashed line is the continuous model. Both curves are in satisfactory agreement with the corresponding experimental points.

We note that the results of generalization of the erosion in the form of U_{er} as a function of g are independent of the selected value of n since the value of n is reduced in such an approximation as a result of calculations. However this procedure leaves the real value of the effective current density unknown if the arc motion is step in nature with a step length of $n > 1$. To determine j it is necessary to measure n independently, using, for example, optical methods. Such a measurement must be carried out with an accuracy of the order of the arc-spot diameter. This imposes strong restrictions on the spatial and temporal resolution of experimental equipment. For example, for a current of 1 kA the diameter of the cathode spot is about 1 mm (see [14, 15]). Applying high-speed optical recording to diagnostics of the movement of the spot with its average velocity of 100 m·sec^{-1}, we must record with a frequency of 100,000 frames per second to obtain a spatial resolution of 1 mm from frame to frame. This is very difficult to implement since, in addition to the high rate of recording, numerous measurements are required because of the random character of distribution of the quantity n and the need for statistical processing of the data obtained with the aim of finding n with a satisfactory accuracy.

CONCLUSIONS

We have proposed a model of erosion of electrodes for the step motion of the arc spot. A comparison of the step and continuous models for the same experimental data demonstrated satisfactory agreement between the two models. Furthermore, we obtained the important parameters of erosion: the effective enthalpy of erosion h_{ef} and the specific microerosion g_0. For the copper cathode of an electric-arc heater these parameters were 66 MJ·kg^{-1} and 3.1 μg·C^{-1} for the continuous-motion model and 81 MJ·kg^{-1} and 2.44 μg·C^{-1} for the step-motion model.

The results of the comparison of the models enable us to infer that whatever the model, the generalizations of experimental data on erosion are similar since both models are based on the fundamentally identical thermophysical approach. Furthermore, it was shown that the real value of the current density in the spot in its step motion can be obtained by thermophysical methods without additional independent diagnostics of the character of movement of the spot. Nonetheless, if a nearly continuous motion of the arc is ensured in the experiments, we can obtain an effective current density rather close to the true value. The current density measured by the thermal method in the cathode spot moving under the action of the magnetic field was $\sim 10^9$ A·m^{-2}.

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NOTATION

B, magnetic induction, T; F, area of the arc spot, m^{-2}; Fo, Fourier number; G, mass rate of erosion (ablation), kg·sec^{-1}; I, arc current, A; L, step length, m; q_0, density of the heat flux supplied from the arc plasma to the spot, W·m^{-2}; U, volt-equivalent of the heat flux in the arc spot, V; U_{er}, volt-equivalent of the erosion heat, V; Q_0, integral heat flux supplied from the arc plasma to the spot, W; $\overline{Q}_{er}$, time-averaged expenditure of heat on erosion, W; Q_{in}, heat flux supplied to the body, W; Q_{out}, heat removal from the material by radiation, W; Q_r, integral heat removal to the electrode by conduction, W; $\overline{Q}_r$, total heat removal to the electrode, averaged over the residence time, W; R, correlation coefficient; T, temperature, K; T_0, surface temperature of the electrode, K; T_f, fusion temperature, K; W and W_s, dimensionless energies of erosion in the models of continuous and step motion respectively; a, thermal diffusivity, $m^2 \cdot sec^{-1}$; b, thickness of the electrode wall, m; d, diameter of the spot, m; f and f_s, dimensionless parameter of fusion in the models of continuous and step motion respectively; g, specific integral erosion, $kg \cdot C^{-1}$; g_0, specific mass micro-erosion, $kg \cdot C^{-1}$; h_{ef}, effective enthalpy of erosion, $J \cdot kg^{-1}$; j, effective current density in the arc spot, $A \cdot m^{-2}$; j_s, imaginary current density for the step motion, $A \cdot m^{-2}$; n, dimensionless length of the arc-spot step, $W \cdot m^{-2}$; q_r, heat-removal density, $W \cdot m^{-2}$; $\overline{q}_r$, heat removal averaged over the time τ_s per unit area of the arc spot, $W \cdot m^{-2}$; s, normalized velocity of the arc, $m \cdot sec^{-1} \cdot A^{-0.5}$; v, average velocity of the arc, $m \cdot sec^{-1}$; λ, thermal conductivity, $W \cdot m^{-1} \cdot K^{-1}$; τ, time, sec; τ_0, time of heating to the fusion temperature, sec; τ_s, time of motionless stay of the spot at a given point of the electrode surface — residence time for the step motion, sec. Subscripts: 0, characteristic value of the quantity (for example, initial temperature T_0, time of heating to the fusion temperate τ_0); ef, effective value; er, erosion; f, fusion, in and out, incoming and outgoing respectively; r, removed; s, parameter for the step motion of the spot.

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